Optimal nonlocal multipartite entanglement concentration based on projection measurements[*]

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We propose an optimal nonlocal entanglement concentration protocol (ECP) for multi-photon systems in a partially entangled pure state, resorting to the projection measurement on an additional photon. One party in quantum communication first performs a parity-check measurement on her photon in an N-photon system and an additional photon, and then she projects the additional photon into an orthogonal Hilbert space for dividing the original N-photon systems into two groups. In the first group, the N parties will obtain a subset of N-photon systems in a maximally entangled state. In the second group, they will obtain some less-entangled N-photon systems which are the resource for the entanglement concentration in the next round. By iterating the entanglement concentration process several times, the present ECP has the maximal success probability which is just equivalent to the entanglement of the partially entangled state. That is, this ECP is an optimal one.

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I. INTRODUCTION

Entanglement plays an important role in quantum information and quantum computation [1]. For example, the powerful speedup of quantum computation resorts to multipartite entanglement [1]. In quantum communication, the two legitimate users, say the sender Alice and the receiver Bob, can use entangled quantum systems to transmit a private key [2, 3]. Moreover, quantum dense coding [4, 5] and quantum teleportation [6] need entangled quantum systems to setup the quantum channel. In a long-distance quantum communication, quantum repeaters are required because quantum signals can only be transmitted over an optical fiber or a free space not more than several hundred kilometers with current technology, although there are some quantum key distribution protocols (QKDs) based on single photons [7] or weak pulses [8–11]. In a practical transmission or the process for storing an entangled quantum system, it inevitably suffers from channel noise and its environment. The noise will make the system decoherent, which will decrease the security of QKD protocols and the fidelity of quantum teleportation and dense coding.

Entanglement purification is used to extract some high-fidelity entangled systems from a less-entangled ensemble in a mixed state and it has been widely studied [12–21]. since Bennett *et al.* [12] proposed the original entanglement purification protocol (EPP) to purify two-photon systems in a Werner state in 1996. For example, Deutsh *et al.* [13] optimized the first EPP with two additional specific unitary operations. In 2001, an EPP based on linear optical elements was introduced by Pan *et al.*[14]. In 2002, Simon and Pan [15] proposed an EPP with a currently available parametric down-conversion (PDC) source. In 2008, an efficient EPP [16] based on a PDC source was introduced with cross-Kerr nonlinearity. In 2010, a two-step deterministic EPP (DEPP) was presented. Subsequently, a one-step DEPP [18] was proposed, only resorting to the spatial entanglement or the frequency entanglement and

linear optical elements. In 2011, Wang *et al.* proposed an interesting EPP for electron spins of quantum dots, resorting to microwave cavity [19] and another EPP for two-photon systems with cross-Kerr nonlinearity [20]. We proposed an efficient multipartite EPP with cross-Kerr nonlinearity in which the cross-combination items can be used to distill some entangled subsystems [21].

Compared with EPPs, entanglement concentration is more efficient for the two remote parties in quantum communication, say Alice and Bob, to distill some maximally entangled systems from an ensemble in a less-entangled pure state because EPPs should consume a great deal of quantum resource as it can only improve the fidelity of systems in a mixed entangled state, not obtain a maximally entangled state directly. Up to now, there are some interesting entanglement concentration protocols [22–28]. For example, Bennett *et al.* [22] proposed the first entanglement concentration protocol (ECP) in 1996 and called it the Schmidt projection method. In 1999, Bose *et al.* [23] proposed another ECP based on entanglement swapping. Subsequently, Shi *et al.*[24] presented a different ECP based on entanglement swapping and a collective unitary evolution. In 2001, Yamamoto *et al.* [25] and Zhao *et al.* [26] proposed an ECP based on polarizing beam splitters (PBSs) independently. Also, they completed its experimental demonstration [29, 30]. In 2008, we proposed an ECP [27] by exploiting cross-Kerr nonlinearities to distinguish the parity of two polarization photons, resorting to the Schmidt projection method. By iteration of the entanglement concentration process, it has a far higher efficiency and yield than those with linear optical elements [25, 26]. In 2010, the first single-photon ECP [28] was discussed with cross-Kerr nonlinearity.

All the existing ECPs [22–28] can be divided into two groups. In the first group, the parameters of the less-entangled pure state $\alpha|H\rangle_A|H\rangle_B + \beta|V\rangle_A|V\rangle_B$ are unknown, such as those in Refs.[22, 25–28]. In the other group, the parameters α and β are known to Alice and Bob [23, 24]. Here $|H\rangle$ and $|V\rangle$ represent the horizontal and the vertical polarizations of photons. The subscripts A and B represent the photons hold by Alice and Bob, respectively. In a practical quantum communication, it is not difficult for Alice and Bob to obtain information about the parameters α and β if they measure an enough number of

sample photon pairs. From the view of efficiency, the ECPs based on a collective unitary evolution [23, 24] are efficient as their success probability equals to a half of the entanglement of the less-entangled pure state $E = min\{2|\alpha|^2, 2|\beta|^2\}$, higher than others [22, 25–28]. However, the collective unitary evolution is usually difficult to implement in the experiment and there are no experimental proposals. Moreover, all existing ECPs [22–28] are, in essence, based on the Schmidt projection method [22] and they exploit a pair of multi-qubit partially entangled systems to obtain a maximally entangled system with the success probability limit E.

In this paper, we proposed an optimal nonlocal ECP for N-photon systems in a known partially entangled pure state, resorting to the projection measurement on an additional photon. It does not depend on a pair of systems in a partially entangled state in each round of concentration, just each system itself and some additional single photons, which makes it far different from others [22–28]. In the present ECP, one of the parties in quantum communication, say Alice first performs a parity-check measurement on her photon A and an additional photon a, and then she projects the additional photon into an orthogonal Hilbert space $\{|\varphi\rangle, |\varphi^\perp\rangle\}$ for dividing the original N-photon systems into two groups. In the first group, the N parties in quantum communication will obtain the N-photon systems in a maximally entangled state when the additional photon is projected into the state $|\varphi^\perp\rangle$. In the second group, they will obtain some N-photon systems in another partially entangled state, which are the resource for entanglement concentration in the next round. By iterating the process several times (usually no more than three times), the present ECP has a success probability P which is nearly equivalent to the entanglement of the partially entangled state E, twice of those based entanglement swapping and a collective unitary evolution [23, 24]. Moreover, it does not require a collective unitary evolution, which decreases the difficulty of its implementation.

II. OPTIMAL NONLOCAL MULTIPARTITE ENTANGLEMENT CONCENTRATION BASED ON PROJECTION MEASUREMENTS

Our ECP is based on a parity-check detector (PCD) and the projection measurement on an additional photon. We first introduce the principle of the PCD based on cross-Kerr nonlinearity below and then our ECP for two-photon systems. In fact, the PCD here is similar to those in Refs.[21, 31].

The Hamiltonian of a cross-Kerr nonlinearity is [31]

$$H_{ck} = \hbar\chi a_s^+ a_s a_p^+ a_p. \qquad (1)$$

Here a_s^+ and a_p^+ are the creation operations, and a_s and a_p are the destruction operations. χ is the coupling strength of the nonlinearity. If a signal state $|\Psi\rangle_s = c_0|0\rangle_s + c_1|1\rangle_s$ ($|0\rangle_s$ and $|1\rangle_s$ denote that there are no photon and one photon respectively in this state) and a coherent probe beam in the state $|\alpha\rangle_p$ couple with a cross-Kerr nonlinearity medium, the evolution of the whole system can be described as:

$$U_{ck}|\Psi\rangle_s|\alpha\rangle_p = e^{iH_{ck}t/\hbar}[c_0|0\rangle_s + c_1|1\rangle_s]|\alpha\rangle_p$$

$$= c_0|0\rangle_s|\alpha\rangle_p + c_1|1\rangle_s|\alpha e^{i\theta}\rangle_p, \qquad (2)$$

where $\theta = \chi t$ and t is the interaction time. The coherent beam picks up a phase shift θ directly proportional to the number of the photons in the Fock state $|\Psi\rangle_s$. Based on this feature of a cross-Kerr nonlinearity, the principle of our PCD is shown in Fig.1. With an X quadrature measurement in which the the states $|\alpha e^{\pm i\theta}\rangle_p$ cannot be distinguished [31, 32], one can distinguish superpositions and mixtures of $|HH\rangle$ and $|VV\rangle$ from $|HV\rangle$ and $|VH\rangle$ as the probe beam $|\alpha\rangle_p$ will pick up a phase shift θ if the two photons is in the state $|HH\rangle_{b_1 b_2}$ or $|VV\rangle_{b_1 b_2}$. If it picks up a phase shift 0, the two photons are in the state $|VH\rangle_{b_1 b_2}$ or $|HV\rangle_{b_1 b_2}$. That is, when the parity of the two photons is odd, the coherent beam will pick up a phase shift 0; otherwise it will pick up a phase shift θ.
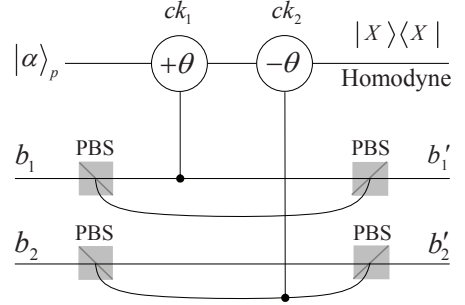

FIG. 1: The principle of a parity-check detector (PCD), the same as that in Ref.[21]. PBS represents a polarizing beam splitter which transmits horizontal polarization $|H\rangle$ and reflects the vertical polarization $|V\rangle$. $\pm\theta$ represent two cross-Kerr nonlinear media which introduce the phase shifts $\pm\theta$ when there is a photon passing through the media. $|X\rangle\langle X|$ represents an X quadrature measurement.
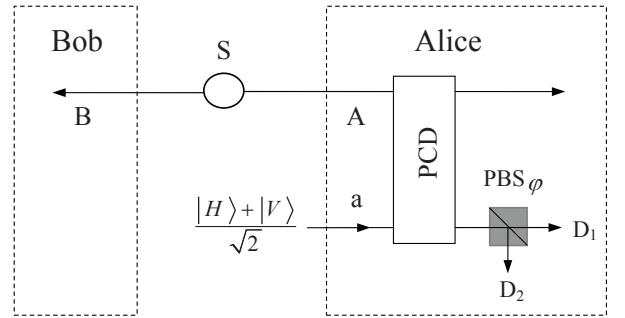

FIG. 2: The schematic diagram of the present entanglement concentration protocol for two-photon systems in a less-entangled pure state. The pair of identical less-entanglement photons A_1 and B_1 are sent to Alice and Bob from source (S), respectively. PCD represents a parity-check detector.

With the PCD shown in Fig.1, the principle of our ECP for two-photon systems in a less-entangled pure state is shown in Fig.2. Suppose the photon pair AB is initially in the following polarization less-entangled pure state:

$$|\Phi_1\rangle_{AB} = \alpha|H\rangle_A|H\rangle_B + \beta|V\rangle_A|V\rangle_B, \qquad (3)$$

where α and β are two real numbers and $|\alpha|^2 + |\beta|^2 = 1$. The same as the ECPs with entanglement swapping [23, 24], Alice and Bob know these two parameters before they distill a subset of maximally entangled photon pairs from a set of photon pairs in the state $|\Phi_1\rangle_{AB}$.

For distilling some maximally entangled photon pairs, Alice prepares an additional photon a in the polarization state $|\Phi\rangle_a = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$ and then performs a parity-check measurement on her photons A and a. If she obtains an even parity, the three-photon system ABa is in the state

$$|\Psi_e\rangle_{ABa} = \alpha|H\rangle_A|H\rangle_B|H\rangle_a + \beta|V\rangle_A|V\rangle_B|V\rangle_a. \quad (4)$$

If she obtains an odd parity, the system is in the state

$$|\Psi_o\rangle_{ABa} = \alpha|H\rangle_A|H\rangle_B|V\rangle_a + \beta|V\rangle_A|V\rangle_B|H\rangle_a, \quad (5)$$

and Alice can transform it into the state $|\Psi_e\rangle_{ABa}$ by performing a bit-flip operation $\sigma_x = |H\rangle\langle V|+|V\rangle\langle H|$ on the photon a. That is, we need only describe the principle of the present ECP when Alice and Bob obtain their photon systems in the state $|\Psi_e\rangle_{ABa}$ below.

We can rewrite the state $|\Psi_e\rangle_{ABa}$ under the orthogonal basis $\{|\varphi_1\rangle_a = \alpha|H\rangle - \beta|V\rangle, |\varphi_1^\perp\rangle_a = \beta|H\rangle + \alpha|V\rangle\}$, that is,

$$\begin{aligned}|\Psi_e\rangle_{ABa} &= (\alpha^2|H\rangle_A|H\rangle_B - \beta^2|V\rangle_A|V\rangle_B)|\varphi_1\rangle_a \\ &+ \sqrt{2}\alpha\beta \cdot \frac{|H\rangle_A|H\rangle_B + |V\rangle_A|V\rangle_B}{\sqrt{2}}|\varphi_1^\perp\rangle_a. \quad (6)\end{aligned}$$

Alice can use a PBS$_\varphi$, whose optical axis is placed at the angle φ_1, and two detectors to complete the measurement on the additional photon a with the basis $\{|\varphi_1\rangle_a, |\varphi_1^\perp\rangle_a\}$, shown in Fig.2. Here $cos\varphi_1 = \alpha$ and $sin\varphi_1 = -\beta$. If Alice obtains the state $|\varphi_1^\perp\rangle_a$ when she measures the additional photon a, the photon pair AB is in the maximally entangled state $|\phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}(|HH\rangle+|VV\rangle)_{AB}$, which takes place with the probability of $2\alpha^2\beta^2$. If Alice obtains the state $|\varphi_1\rangle_a$, the photon pair AB is in another partially entangled pure state (without normalization)

$$|\Phi_2\rangle_{AB} = \alpha^2|H\rangle_A|H\rangle_B - \beta^2|V\rangle_A|V\rangle_B, \quad (7)$$

which takes place with the probability of $\alpha^4 + \beta^4 = 1 - 2\alpha^2\beta^2$.

It is obvious that the less-entangled pure state $|\Phi_2\rangle_{AB}$ has the same form as the state $|\Phi_1\rangle_{AB}$ shown in Eq.(3). We need only replace α and β with $\alpha' \equiv \frac{\alpha^2}{\sqrt{\alpha^4+\beta^4}}$ and $\beta' \equiv -\frac{\beta^2}{\sqrt{\alpha^4+\beta^4}}$, respectively. That is, Alice and Bob can distill the maximally entangled state $|\phi^+\rangle_{AB}$ from the state $|\Phi_2\rangle_{AB}$ with the probability of $2(\alpha^4+\beta^4)\alpha'^2\beta'^2$ by adding another additional photon a_1 and a parity-check measurement. Moreover, they can distill the photon pairs in the maximally entangled state $|\phi^+\rangle_{AB}$ from the less-entangled systems in the next round yet. That is, by iterating the entanglement concentration process n times, the total success probability of this ECP is

$$\begin{aligned}P_n &= 2[\frac{\alpha^2\beta^2}{\alpha^2 + \beta^2} + \frac{\alpha^4\beta^4}{(\alpha^2 + \beta^2)(\alpha^4 + \beta^4)} \\ &+ \frac{\alpha^8\beta^8}{(\alpha^2 + \beta^2)(\alpha^4 + \beta^4)(\alpha^8 + \beta^8)}\end{aligned}$$
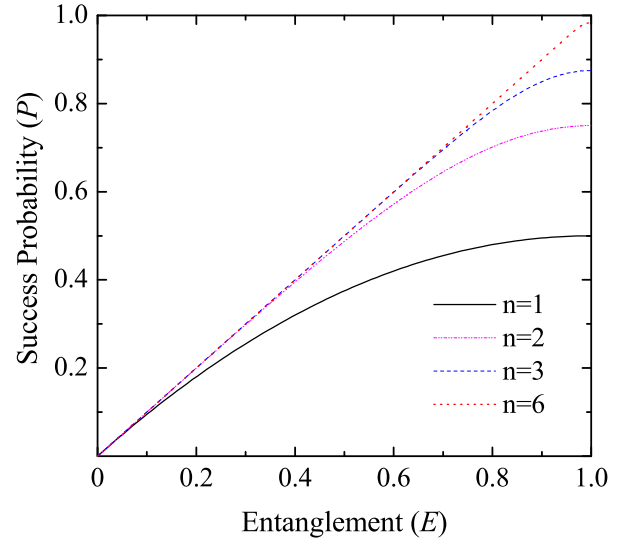

FIG. 3: (Color online) The relation between the success probability of the present ECP P and the entanglement of the partially entangled state E under the iteration numbers of entanglement concentration $n = 1, 2, 3,$ and 6, respectively.

$$\begin{aligned}&+ \frac{\alpha^{16}\beta^{16}}{(\alpha^2 + \beta^2)(\alpha^4 + \beta^4)(\alpha^8 + \beta^8)(\alpha^{16} + \beta^{16})} + \cdots \\ &+ \frac{\alpha^{2^n}\beta^{2^n}}{(\alpha^2 + \beta^2)(\alpha^4 + \beta^4)(\alpha^8 + \beta^8)\cdots(\alpha^{2^n} + \beta^{2^n})}]. \quad (8)\end{aligned}$$

If $|\alpha| \leq |\beta|$, the entanglement of the state $|\Phi_1\rangle_{AB} = \alpha|H\rangle_A|H\rangle_B + \beta|V\rangle_A|V\rangle_B$ is $E = 2|\alpha|^2$ [23, 24]. The relation between the success probability P that the two parties obtain a photon pair AB in the maximally entangled state $|\phi^+\rangle_{AB}$ from a pair in the partially entangled state $|\Phi_1\rangle_{AB}$ and the entanglement E is shown in Fig.3. When $E < 0.4$, Alice and Bob need only perform the entanglement concentration process twice ($n = 2$) for obtaining the success probability P nearly equivalent to the entanglement E. When $0.4 < E < 0.72$, they should perform the process three times for obtaining an optimal success probability. From Fig.3, one can see that six times for the iteration of the entanglement concentration process is enough to obtain an optimal success probability.

III. DISCUSSION AND SUMMARY

It is straightforward to generalize our ECP to reconstruct maximally entangled N-photon GHZ states from partially entangled GHZ-class states. Suppose the partially entangled N-photon GHZ-class states are described as follows:

$$|\Phi_N\rangle = \alpha|HH\cdots H\rangle_{AB...Z} + \beta|VV\cdots V\rangle_{AB...Z}, \quad (9)$$

where $|\alpha|^2 + |\beta|^2 = 1$. The subscript $A, B, \ldots,$ and Z represent the photons held by Alice, Bob, ..., and Zach, respectively. If we define $|H'\rangle_{N'} \equiv |H\cdots H\rangle_{B...Z}$ and $|V'\rangle_{N'} \equiv |V\cdots V\rangle_{B...Z}$, the

state $|\Phi_N\rangle$ can be rewritten as

$$|\Phi_N\rangle = \alpha|H\rangle_A|H'\rangle_{N'} + \beta|V\rangle_A|V'\rangle_{N'}. \qquad (10)$$

It has the same form as the state $|\Phi_1\rangle_{AB}$ shown in Eq.(3). So the N parties can also obtain the maximally entangled N-photon systems with the total success probability P if Alice deals with the photon A in the N-photon system and another additional photon a_2 in the same way as the case with non-maximally entangled two-photon pure state.
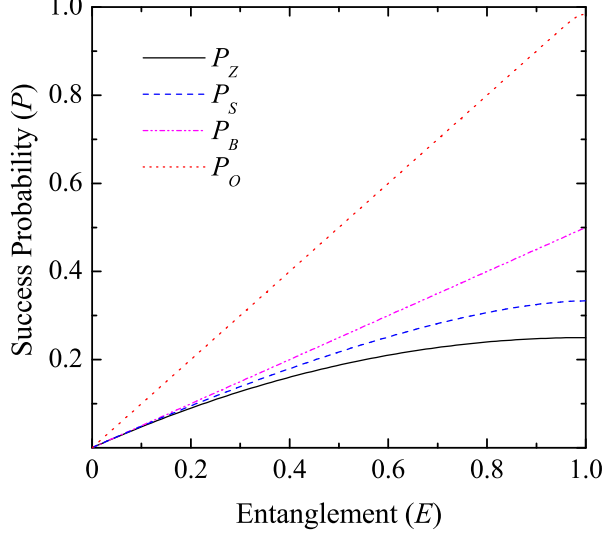

FIG. 4: (Color online) The comparison of the success probability between the present ECP and other ECPs. P_O is the success probability in the present ECP. P_Z, P_S, and P_B are the success probabilities for each system in the works by Zhao *et al.* [26] and Yamamoto *et al.* [25], Sheng *et al.* [27], and Bose *et al.* [23] and Shi *et al.* [24], respectively.

The comparison of the success probabilities between the present ECP and other ECPs [22–28] is shown in Fig.4. Here P_Z represents the success probability in the ECP by Zhao *et al.* [26] and Yamamoto *et al.* [25]. It is just the one in the ECP based on the Schmidt projection method by Bennett *et al.* [22]. P_O and P_S represent the success probability in the present ECP and that in the ECP with cross-Kerr nonlinearity [27] under the iteration number $n = 6$, respectively. P_B is the success probability in the ECPs by Bose *et al.* [23] and Shi *et al.* [24] in an ideal condition. All other existing ECPs [22–28] are based on the Schmidt projection method although some exploit a collective unitary evolution to improve their success probability, which requires at least a pair of systems in a partially entangled state to distill a system in a maximally entangled state with some probability. For each system in a partially entangled original state, the maximal success probability is just a half of the entanglement of the initial state E. However, the present ECP does not depend on a pair of systems in a partially entangled state in each round of concentration, just each system itself and some additional single photons, which makes it far different from others [22–28]. Moreover, it is obvious that the present ECP is far more efficient

than others as its success probability equals to the entanglement of the partially entangled state. That is, the present ECP is an optimal one.

The present ECP requires that the parties obtain the information about the initial state, as the same as those in Refs.[23, 24], but different from those in Refs.[22, 25–27]. On one hand, those ECPs [22, 25–27], which do not require the parties know accurately the information about the initial state, can be used to concentrate nonlocally the systems in a partially entangled known state. Especially, the ECP based on linear optical elements [25, 26] and the efficient ECP based on nonlinear optics [27] give a detailed way for its implementation, which are different from the ones based on a collective unitary evolution. On the other hand, all the existing ECPs [22–28] are, in essence, based on the Schmidt projection method in which a two-photon system in the state $\alpha|H\rangle_{A_1}|H\rangle_{B_1} + \beta|V\rangle_{A_1}|V\rangle_{B_1}$ and another one in the state $\beta|H\rangle_{A_2}|H\rangle_{B_2} + \alpha|V\rangle_{A_2}|V\rangle_{B_2}$ are used to distill a two-photon system in the state $\frac{1}{\sqrt{2}}(|H\rangle|H\rangle + |V\rangle|V\rangle)_{A_1 B_2}$ with an average success probability of $|\alpha\beta|^2$ or $|\alpha|^2$ (if $|\alpha| \leq |\beta|$) for each system.

The key element in the present ECP is the PCD. We construct the PCD with cross-Kerr nonlinearity. At present, the implementation of a clean cross-Kerr nonlinearity is still difficult in the experiment, especially with natural cross-Kerr nonlinearities. Fortunately, the PCD in our ECP does not require a large nonlinearity and it works for small values of the cross-Kerr coupling, which decreases the difficulty of its implementation [31, 32]. On the one hand, the fidelity of the PCD with cross-Kerr nonlinearity in a practical application at present can not be improved to be a unit as there are always phase noises [33–35]. On the other hand, a great number of works are focused on the photon-photon nonlinear interaction [36, 37], which provides many ways for constructing the PCD, such as these based on quantum dot spins in microwave cavity [38, 39], a Rydberg atom ensemble [40], a cavity waveguide [41], hollow-core waveguides [42], and so on. We use the PCD based on cross-Kerr nonlinearity to describe the principle of our ECP. It works with the PCDs based on other nonlinear interactions.

In summary, we have proposed an optimal nonlocal ECP for multipartite partially entangled states, resorting to projection measurements. Alice exploits the PCD based on cross-Kerr nonlinearity to extend the partially entangled N-photon system to an $(N + 1)$-photon system first and then she projects the additional photon with a suitable orthogonal basis. By detecting the state of the additional photon, the N-parties in quantum communication can divide their N-photon systems into two groups. One is in the maximally entangled state and the other is in another partially entangled state which is just the resource for the entanglement concentration in the next round. By iterating the entanglement concentration process several times, the N parties can obtain a subset of N-photon systems in the maximally entangled state with the maximal success probability which is just equivalent to the entanglement of the partially entangled state. Compared with other ECPs [22–28], the present ECP has the optimal success probability, without resorting to a collective unitary evolution [24].

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